1. Unfinished business. Finish the proof of $\Gamma \vdash \mathsf{mfredrik says read}('15316\text{-grades.xlsx'})$ from the following assumptions.

$$\begin{array}{l} Q_1 \equiv \texttt{isKey}(\texttt{ca},\texttt{pk}_{\texttt{ca}}) \\ Q_2 \equiv \texttt{sign}_{\texttt{sk}_{\texttt{ca}}}(\texttt{isKey}(\texttt{tpm},\texttt{pk}_{\texttt{tpm}})) \\ Q_3 \equiv \texttt{sign}_{\texttt{sk}_{\texttt{tpm}}}(\texttt{isKey}(\texttt{os},\texttt{pk}_{\texttt{os}})) \\ Q_4 \equiv \texttt{sign}_{\texttt{sk}_{\texttt{mfredrik}}}(\forall x.(\texttt{os says read}(x)) \rightarrow (\texttt{mfredrik says read}(x))) \\ Q_5 \equiv \texttt{sign}_{\texttt{sk}_{\texttt{os}}}(\texttt{read}(\texttt{'15316}\texttt{-}\texttt{grades}.\texttt{xlsx'})) \\ Q_6 \equiv \forall x.(\texttt{tpm says isKey}(x,\texttt{pk}_x)) \rightarrow \texttt{isKey}(x,\texttt{pk}_x) \\ Q_7 \equiv \texttt{isCA}(\texttt{ca}) \end{array}$$

Note: The lecture notes did not mention the assumption $isKey(mfredrik, pk_{mfredrik})$, which you can use in your assumptions and denote Q_8 .

2. Countersigning. It is common practice in PKI to have the CA issue weaker certificates that rely on a *countersignature* for verification. So suppose that ca is the certificate authority and cs is the countersigner. Then ca might issue a certificate to cmu that consists of the following.

$$sign_{sk_{cmu}}(cs \ says \ isKey(cmu, pk_{cmu}) \rightarrow isKey(cmu, pk_{cmu}))$$
(1)

Then cs must issue a second certificate, which comes with an expiration date.

$$sign_{sk_{cs}}(isbefore(exp) \rightarrow isKey(cmu, pk_{cmu}))$$
(2)

Part 1. Explain how one can verify the authenticity of pk_{cmu} from (1) and (2), along with assumptions $\Gamma = isCA(ca)$, $isKey(cs, pk_{cs})$, isbefore(exp).

Part 2. Explain how countersigning can mitigate the effects of key compromise. In particular, describe the consequences of **cmu**'s private key being compromised if the corresponding public key is certified in this way, and how they are less severe than if **cmu** had obtained a certificate directly from **ca**. Then describe the consequences of **cs**'s signing key being compromised, and why this is less severe than if **ca**'s signing key is compromised.