

Assignment 5

Proof-Carrying Authorization

15-316: Software Foundations of Security & Privacy

Due Tue, Mar 24, 2026

85 points

Your solution should be handed in as a file `hw5.pdf` to Gradescope. If at all possible, write your solutions in \LaTeX . The handout `hw5-pca.zip` includes the \LaTeX sources for some lectures and the necessary style files which provide some examples for rules, derivations, and proofs. You may use up to two late days as usual.

1 Authorization Logic [25 points]

For each of the following, either give a derivation in the sequent calculus or give a counterexample. In the latter case, use p and q as atomic formulas and demonstrate that the counterexample is not derivable.

Task 1 (5 pts) $(A \text{ says } P) \wedge (A \text{ says } Q) \vdash A \text{ says } (P \wedge Q)$

Task 2 (5 pts) $A \text{ says } (P \wedge Q) \vdash (A \text{ says } P) \wedge (A \text{ says } Q)$

Task 3 (5 pts) $(A \text{ says } P) \vee (A \text{ says } Q) \vdash A \text{ says } (P \vee Q)$

Task 4 (5 pts) $A \text{ says } (P \vee Q) \vdash (A \text{ says } P) \vee (A \text{ says } Q)$

Task 5 (5 pts) $A \text{ says } \forall x. P(x) \vdash \forall x. A \text{ says } P(x)$

2 Trust [30 points]

We informally define that A **trusts** B if whenever B affirms P then A also affirms P . Unfortunately, we cannot express this directly in our authorization logic because the right-hand side of the definition

$$A \text{ trusts } B \triangleq \forall P. B \text{ says } P \rightarrow A \text{ says } P$$

quantifies over all possible propositions. In authorization logic, quantifiers only range over principals and constants from some finite domain (like rooms or files or homework assignments, etc.).

Instead we assume that there is a *fixed preorder of principals* where $A \leq B$ means that A trusts B . We assume that this relation is reflexive and transitive.

Task 6 (15 pts) Define rules in the sequent calculus for authorization logic that incorporate the trust relationship. Your premises may directly refer to $A \leq B$ for principals A and B . You may modify existing rules *says R* , *says L* , and *aff*, and you may add new rules. Other inference rules should remain unchanged.

Task 7 (5 pts) Prove $A \text{ says } P \wedge B \text{ says } Q \vdash A \text{ says } (P \wedge Q)$ in your system, provided $A \leq B$.

Task 8 (10 pts) Give a counterexample to $A \text{ says } P \wedge B \text{ says } Q \vdash B \text{ says } (P \wedge Q)$ provided $B \not\leq A$. You should be able to use your rules to prove that the counterexample cannot be derived. If not, explain for partial credit why it is difficult or impossible.

3 Groups and Access Control [30 points]

In the AFS file system used on the `linux.andrew` machines there are two kinds of principals: individuals (identified by their Andrew id) and groups (identified by a name distinct from an individual id). We formalize a variant of the Andrew file system access control with the following policies:

- (i) Any principal may create a new group they own simply by saying so. Assume that adding such an affirmation to the policy would fail if the group already exists.
- (ii) The owner of a group is always one of its members.
- (iii) Only the owner of a group may add members to it. We are not concerned with revoking group membership.
- (iv) The owner of a file may grant a principal read or write access simply by saying so.
- (v) If a group has read or write access to a file, each member of the group has read or write access, respectively, to the file.

In order to formalize this policy in authorization logic, we use the following vocabulary:

- Principals A, B, C, \dots could denote an individual or a group.
- Groups G, \dots if we know a principal must be a group.
- *admin*. The principal who administers policy, affirming rules for groups and access control. Depending on the configuration, *admin* could be a group or an individual.
- Mode M , which is either *read* or *write*.
- File names F .
- $\text{ownsGroup}(A, G)$. Principal A owns group G .
- $\text{ownsFile}(A, F)$. Principal A owns file F .
- $\text{member}(A, G)$. Principal A is a member of group G .
- $\text{mayOpen}(A, F, M)$. Principal A may open file F in mode M .

Task 9 (10 points) Write out the access control policy explained above in authorization logic. Label your affirmations so they can be referenced in later tasks.

Task 10 (10 points) Write out the necessary affirmations by *mfredrik* or *admin* to express each of the following. Label your affirmations so they can be referenced in later tasks.

1. Individual *mfredrik* owns group *316_ta* and files *316_roster* and *316_grades*.
2. Individuals *kman* and *jzou* are members of *316_ta*.
3. Group *316_ta* may read file *316_roster* and read and write file *316_grades*.

Task 11 (10 points) Suppose we add the following affirmation to the policy:

kman **says** member(*studentA*, *316_ta*)

1. Can we derive that *studentA* may read file *316_roster*? Explain why or why not, referring to specific policy rules from the previous tasks.
2. What affirmation(s) would need to be added to authorize *studentA* to read *316_roster*?